Optimal initial values
and regularity conditions of Besov space type
for weak solutions to the Navier-Stokes system

Reinhard Farwig (Technische Universität Darmstadt)

`farwig@mathematik.tu-darmstadt.de`

In this talk we present recent results jointly obtained with Hermann Sohr (Paderborn) and W. Varnhorn (Kassel).

The first question concerns the *optimal condition* on initial values $u_0 \in L^2_\sigma(\Omega)$ to get a locally regular solution u in Serrin's class $L^s(0, T; L^q(\Omega))$ (with $\frac{2}{s} + \frac{3}{q} = 1$, $s > 2$, $q > 3$) to the instationary Navier-Stokes system in a bounded domain $\Omega \subset \mathbb{R}^3$. It is shown that the condition

$$\int_0^\infty \|e^{-\tau A} u_0\|_{L^q(\Omega)}^s \, d\tau < \infty \tag{1}$$

is *necessary and sufficient* for this local in time regularity result; here A denotes the Stokes operator on $L^2_\sigma(\Omega)$. Condition (1) is weaker than the more classical assumptions $u_0 \in \mathcal{D}(A^{1/4})$ or $u_0 \in L^3(\Omega)$ and ensures that the solution $e^{-\tau A} u_0$ of the linear Stokes problem lies in Serrin's class $L^s(0, \infty; L^q(\Omega))$. It can be rewritten in the form

$$u_0 \in \mathbb{B}_{q,s}^{-2/s}(\Omega)$$

where $\mathbb{B}_{q,s}^{-2/s}(\Omega)$ denotes a solenoidal subspace of the usual Besov space $B_{q,s}^{-2/s}(\Omega)$.

Actually, it suffices to consider the integral in (1) on a finite time interval $(0, \delta)$, leading to a Besov space $\mathbb{B}_{q,s;(\delta)}^{-2/s}(\Omega)$ with equivalent norm. Using these spaces we find new regularity and uniqueness criteria for weak solutions.

[1] R. Farwig, H. Sohr and W. Varnhorn: *Optimal initial value conditions for the existence of local strong solutions of the Navier-Stokes squations.* Math. Ann. 345 (2009), 631-642

[2] R. Farwig, H. Sohr and W. Varnhorn: *Extensions of Serrin's uniqueness and regularity conditions for the Navier-Stokes equations.* J. Math. Fluid Mech. 14 (2012), 529-540

[3] R. Farwig, H. Sohr and W. Varnhorn: *Besov space regularity conditions for weak solutions of the Navier-Stokes equations.* Technische Universität Darmstadt, FB Mathematik, Preprint no. 2669 (2103), submitted

[4] R. Farwig: *On regularity of weak solutions to the instationary Navier-Stokes system - a review on recent results.* Ann. Univ. Ferrara (to appear)