Mathias-Prikry type forcing and dominating real

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joint work with
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Introduction

Destructibility of ideals by Forcing

Let $\mathcal{I}$ be an ideal on a countable set X.
We say $\mathcal{I}$ is **tall** if $\forall A \in [X]^{\omega} \exists I \in \mathcal{I}(|A \cap I| = \aleph_0)$.

Example

If $\mathcal{A}$ is a mad family, then

$$\mathcal{I}(\mathcal{A}) = \{I \subset \omega : \exists \text{ finite } \mathcal{F} \subset \mathcal{A}(I \subset \cup\mathcal{F})\}$$

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Definition

Given an ideal $\mathcal{I}$ and a forcing notion $\mathbb{P}$. $\mathbb{P}$ destroys $\mathcal{I}$ if there exists a $\mathbb{P}$-name $\dot{A}$ for an infinite subset of ω such that

$$\Vdash_{\mathbb{P}} \forall I \in \mathcal{I} \cap V\left(\left|\dot{A} \cap I\right| < \aleph_0\right).$$

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Question

When does a given forcing destroy a given ideal?

Forcing quotients

Let $\mathcal{I}$ be a σ-ideal on a Polish space X. Let $\mathbb{P}_{\mathcal{I}}$ be a forcing notion of $\mathcal{I}$-positive Borel subsets of X, ordered by inclusion.

Many proper forcing notions can be presented as forcing of the form $\mathbb{P}_{\mathcal{I}}$.

Example

$\mathbb{C} \simeq \mathbb{P}_{\mathbf{meager}} = \mathcal{B}(2^\omega)/\mathcal{M}$, $\mathbb{B} = \mathbb{P}_{\mathbf{null}} = \mathcal{B}(2^\omega)/\mathcal{N}$,
$\mathbb{S} = \mathcal{B}(2^\omega)/\mathbf{cntble}$, $\mathbb{L} = \mathcal{B}(\omega^\omega)/\mathbf{not-dominating}, \ldots$

trace ideal and Katětov order

Definition (Brendle)

Let $\mathcal{I}$ be an σ-ideal on $\mathbf{2}^\omega$(or ω^ω). Its trace ideal $\mathbf{tr}(\mathcal{I})$ is an ideal on $\mathbf{2}^{<\omega}$ (or $\omega^{<\omega}$) defined by

$$a \in \mathbf{tr}(\mathcal{I}) \text{ iff } \{r : \exists^\infty n \in \omega(r \upharpoonright n \in a)\} \in \mathcal{I}.$$

Definition

Let X and Y are countable. Let $\mathcal{I}$ be an ideal on X and $\mathcal{J}$ be an ideal on Y.

$$\mathcal{I} \leq_K \mathcal{J} \quad \text{if} \quad \exists f : Y \to X \forall I \in \mathcal{I}(f^{-1}[I] \in \mathcal{J}).$$

We call this order $\leq_K$ Katětov order.

trace ideal and CRN

Let $\mathcal{J}$ be an ideal on a countable set and $X \in \mathcal{J}^+$. Then

$$\mathcal{J} \restriction X = \{J \cap X : J \in \mathcal{J}\}$$

is an ideal on X.

Theorem (Hrušák, Zapletal)

If $\mathbb{P}_I$ is a proper forcing with CRN and $\mathcal{J}$ is an ideal on ω, then the following are equivalent:

1. *there is a $B \in \mathbb{P}_I$ such that $B \Vdash$ "$\mathcal{J}$ is destroyed".*
2. *there is a $\mathbf{tr}(I)$-positive set a such that $\mathcal{J} \leq_K \mathbf{tr}(I) \restriction a$.*

Question

Fix an ideal $\mathcal{I}$. Is there a forcing notion which destroys $\mathcal{I}$ and have a nice property?

Theorem (Laflamme)

Every $\mathbf{F}_\sigma$ ideal can be destroyed by a proper ω^ω-bounding forcing.

Laver forcing associated with filters

Definition

Let $\mathcal{I}$ be an ideal on ω.

$$T \in \mathbb{L}_{\mathcal{I}^*} \text{ if } T \subset \omega^\omega \text{ is tree} \land$$
$$\forall t \in T(\mathrm{stem}(T) \subset t \to \mathrm{Succ}_T(t) = \{n \in \omega : t^\frown n \in T\} \in \mathcal{I}^*)).$$

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Proposition

Let G be a $\mathbb{L}_{\mathcal{I}^*}$-generic and $\dot{\ell}_G$ and $\dot{a}_G$be $\mathbb{L}_{\mathcal{I}^*}$-names such that

$$\Vdash \dot{\ell}_G = \bigcup\{\mathrm{stem}(T) : T \in G\} \in \omega^\omega \land \dot{a}_G = \mathrm{rng}(\dot{\ell}_G) \in [\omega]^\omega$$

Then

$$\Vdash \forall f \in \omega^\omega \cap V(f \leq^* \dot{\ell}_G) \text{ and}$$
$$\forall I \in \mathcal{I} \cap V \forall X \in \mathcal{I}^+ \cap V(|I \cap \dot{a}_G| < \aleph_0 \land |\dot{a}_G \cap X| = \aleph_0).$$

Mathias forcing associated with filters

Definition

Let $\mathcal{I}$ be an ideal on ω. Then

$$\langle s, F\rangle \in \mathbb{M}_{\mathcal{I}^*} \text{ if } s \in [\omega]^{<\omega} \wedge F \in \mathcal{I}^* \wedge s \cap F = \emptyset$$

ordered by

$$\langle s, F\rangle \leq \langle t, G\rangle \text{ if } s \supset t \wedge F \subset G \wedge s \setminus t \subset G.$$

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Proposition

Let G be a $\mathbb{M}_{\mathcal{I}^*}$-generic and $\dot{A}_G$ be a $\mathbb{M}_{\mathcal{I}^*}$-name such that

$$\Vdash \dot{A}_G = \bigcup\{s \in [\omega]^{<\omega} : \langle s, H\rangle \in G\}.$$

Then

$$\Vdash \forall I \in \mathcal{I} \forall X \in \mathcal{I}^+ (|\dot{A}_G \cap I| < \aleph_0 \wedge |\dot{A}_G \cap X| = \aleph_0).$$

Preservation of countably tallness

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Theorem

Let $\mathcal{I}$ be an ideal on ω. Then

1. *(Błaszczyk-Shelah) $\mathbb{M}_{\mathcal{I}^*}$ does not add a Cohen real if and only if $\mathcal{I}^*$ is a selective ultrafilter.*
2. *$\mathbb{L}_{\mathcal{I}^*}$ does not add a Cohen real if and only if $\mathcal{I}^*$ is a nowhere dense ultrafilter.*

Theorem

1. *(Canjar) If $\mathcal{U}$ is either rapid ultrafilter or not a P-point ultrafilter, then $\mathbb{M}_{\mathcal{U}}$ adds a dominating real.*
2. *(Brendle) If $\mathcal{I}$ is $\mathbf{F}_{\sigma}$-ideal, then $\mathbb{M}_{\mathcal{I}^*}$ doesn't add any dominating real.*

Theorem (Brendle, Hrušák)

Let $\mathcal{I}$ be an ideal on ω. Then the following are equivalent:

1. $\forall X \in \mathcal{I}^+ \forall \mathcal{J} \leq_K \mathcal{I} \restriction X$ ($\mathcal{J}$ *is not countably tall.*)
2. $\mathbb{L}_{\mathcal{I}^*}$ *preserves countable tallness, i.e., if* $\mathcal{L}$ *is countable tall, then* $V^{\mathbb{L}_{\mathcal{I}^*}} \models \mathcal{L}$ *is countable tall.*
3. $\mathbb{L}_{\mathcal{I}^*}$ *strongly preserves countable tallness, i.e.,*

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3. $\mathbb{L}_{\mathcal{I}^*}$ *strongly preserves countable tallness, i.e., for every* $\mathbb{L}_{\mathcal{I}^*}$*-names* $\{\dot{A}_n : n \in \omega\}$ *such that* $\Vdash \forall n \in \omega(\dot{A}_n \in [\omega]^\omega)$*, there exists* $\{A_n : n \in \omega\} \subset [\omega]^\omega$ *in* V *such that*

$$\text{if } X \in [\omega]^\omega \cap V \text{ satisfies } \forall n \in \omega(|A_n \cap X| = \aleph_0), \\ \text{then } \Vdash \forall n \in \omega(|X \cap \dot{A}_n| = \aleph_0). \tag{1}$$

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For an ideal $\mathcal{I}$ on ω,

$$\mathcal{I}^{<\omega} = \{A \subset [\omega]^{<\omega} \setminus \{\emptyset\} : \exists I \in \mathcal{I} \forall a \in A (a \cap I \neq \emptyset)\}.$$

Then $\mathcal{I}^{<\omega}$ is an ideal on $[\omega]^{<\omega} \setminus \{\emptyset\}$.

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Then $\mathcal{I}^{<\omega}$ is an ideal on $[\omega]^{<\omega} \setminus \{\emptyset\}$.

Theorem (Hrušák, Minami)

The followings are equivalent.

1. $\forall X \in (\mathcal{I}^{<\omega})^{+} \forall \mathcal{J} \leq_K \mathcal{I}^{<\omega} \restriction X (\mathcal{J}$ *is not countable tall.*$)$
2. $\mathbb{M}_{\mathcal{I}^*}$ *strongly preserves countably tallness.*
3. $\mathbb{M}_{\mathcal{I}^*}$ *preserves countably tall family.*

$\mathbb{M}_{\mathcal{I}^*}$ and dominating real

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Theorem (Hrušák, Minami)

The following are equivalent.

1. $\mathbb{M}_{\mathcal{I}^*}$ *adds a dominating real.*
2. $\mathcal{I}^{<\omega}$ *is not* P^+ *ideal.*

Definition

$\mathcal{J}$ is P^+-ideal if for every decreasing sequence $\{X_n : n \in \omega\}$ of $\mathcal{J}$-positive set, there exists $X \in \mathcal{J}^+$ such that $X \subset^* X_n$.

From (1) to (2).
Let $\dot{g}$ be a $\mathbb{M}_{\mathcal{I}^*}$-name for a dominating real, i.e.,

$$\forall f \in \omega^\omega \cap V (\Vdash f \leq^* \dot{g}).$$

For $f \in \omega^\omega \cap V$, there exists $s_f \in [\omega]^{<\omega}$, $F_f \in \mathcal{I}^*$ and $n_f \in \omega$ such that

$$\langle s_f, F_f \rangle \Vdash \forall m \geq n_f (f(n) \leq \dot{g}(m)).$$

Fix $s \in [\omega]^{<\omega}$ and $n \in \omega$ such that

$$\mathcal{F} = \{f \in \omega^\omega : s_f = s \wedge n_f = n\}$$

is a dominating family.
Define

$$X_s = \{t \in [\omega \setminus \max(s)]^{<\omega} : \exists F \in \mathcal{I}^* \exists m \geq n \\ (\langle s \cup t, F \rangle \text{ decides } \dot{g}(m))\}.$$

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Claim

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Let $z_t = \{m \geq n : \exists F \in \mathcal{I}^* (\langle s \cup t, F \rangle \text{ decides } \dot{g}(m))\}$.
Then define $H : X_s \to \omega$ by

$$H(t) = \begin{cases} \max(z_t) & \text{if } |z_t| < \omega \\ \min(z_t \setminus \max(t)) & \text{otherwise.} \end{cases}$$

Let $Y_m = \{H^{-1}[\omega \setminus m]\}$ for $m \geq n$. Then $Y_{m+1} \subset Y_m$.

Claim

$Y_m \in (\mathcal{I}^{<\omega})^+$ for $m \geq n$.

Let $Y \subset^* Y_m$ for $m \geq n$. We shall show $Y \in \mathcal{I}^{<\omega}$.
Assume to the contrary that $Y \in (\mathcal{I}^{<\omega})^+$. Define a function $g : \omega \to \omega$ by

$$g(m) = \begin{cases} \min\{k : \exists t \in Y \exists m \in \omega \exists F \in \mathcal{I}^* \\ \qquad (H(t) = m \wedge \langle s \cup t, F\rangle \Vdash \dot{g}(m) = k)\} \\ \text{ if there exists } t \in Y \text{ such that } H(t) = m \\ \\ 0 \\ \text{ otherwise.} \end{cases}$$

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Since $\mathcal{F}$ is a dominating family, there exists $f \in \mathcal{F}$ and $m_0 \geq n$ such that $\forall m \geq m_0(g(m) \leq f(m))$.

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Then there exists $m \geq m_0$ and $t \in Y \cap Y_m \cap F_f$. Also we can find $F \in \mathcal{I}^*$ such that $\langle s \cup t, F \rangle \Vdash \dot{g}(m) = g(m)$.

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Since $\mathcal{F}$ is a dominating family, there exists $f \in \mathcal{F}$ and $m_0 \geq n$ such that $\forall m \geq m_0 (g(m) \leq f(m))$.
Then there exists $m \geq m_0$ and $t \in Y \cap Y_m \cap F_f$. Also we can find $F \in \mathcal{I}^*$ such that $\langle s \cup t, F\rangle \Vdash \dot{g}(m) = g(m)$.
However $\langle s, F_f\rangle \Vdash$ "$\forall m \geq n(f(m) < \dot{g}(m))$" and $\langle s \cup t, F\rangle$ is compatible with $\langle s, F_f\rangle$. It is contradiction. Therefore $Y \in \mathcal{I}^{<\omega}$ and $\mathcal{I}^{<\omega}$ is not P^+-ideal.

From (2) to (1). Let $\langle X_n : n \in \omega \rangle \subset (\mathcal{I}^{<\omega})^+$ such that

1. $X_{n+1} \subset X_n$ for $n \in \omega$ and
2. there is no pseudointersection in $(\mathcal{I}^{<\omega})^+$.

Let $\langle a_k : k \in \omega \rangle$ be an enumeration of $[\omega]^{<\omega} \setminus \{\emptyset\}$.
Let $\dot{A}_{gen}$ be a $\mathbb{M}_{\mathcal{I}^*}$-name for $\mathbb{M}_{\mathcal{I}^*}$-generic real($\subset \omega$).

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Let $\dot{A}_{gen}$ be a $\mathbb{M}_{\mathcal{I}^*}$-name for $\mathbb{M}_{\mathcal{I}^*}$-generic real($\subset \omega$).
Define $\mathbb{M}_{\mathcal{I}^*}$-name $\dot{g}$ for a function from ω to ω by

$$\Vdash \dot{g}(n) = \min\{k : a_k \subset [\dot{A}_{gen}]^{<\omega} \cap X_n \wedge \max(\bigcup\{a_m : l < n \wedge m = \dot{g}(l)\}) < \min(a_k)\}.$$

We shall show $\dot{g}$ be a dominating real.

Let $f \in \omega^\omega \cap V$ and $\langle s, F \rangle \in \mathbb{M}_{\mathcal{I}^*}$.

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Let

$$I_f = \{a_k \in [\omega]^{<\omega} \setminus \{\emptyset\} : \exists n \in \omega(a_k \in X_n \wedge k \leq f(n))\}.$$

Then $I_f \subset^* X_n$ for every $n \in \omega$. So $I_f \in \mathcal{I}^{<\omega}$ by definition of X_n.
Let $\hat{I}_f \in \mathcal{I}$ such that $\forall a \in I_f(a \cap \hat{I}_f \neq \emptyset)$. Then $F \setminus \hat{I} \in \mathcal{I}^*$ and $[F \setminus \hat{I}]^{<\omega} \cap I_f = \emptyset$.

Let $f \in \omega^\omega \cap V$ and $\langle s, F\rangle \in \mathbb{M}_{\mathcal{I}^*}$.
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Let $\hat{I}_f \in \mathcal{I}$ such that $\forall a \in I_f(a \cap \hat{I}_f \neq \emptyset)$. Then $F \setminus \hat{I} \in \mathcal{I}^*$ and $[F \setminus \hat{I}]^{<\omega} \cap I_f = \emptyset$.
Put $|s|$=m. Then $\langle s, F \setminus I\rangle \leq \langle s, F\rangle$ and

$$\langle s, F \setminus I\rangle \Vdash \forall n > m(f(n) < \dot{g}(n)).$$

Borel case

Theorem

Suppose $\mathcal{I}$ is a Borel ideal. Then the following are equivalent.

1. *$\mathcal{I}$ can be extended to an ideal $\mathcal{J}$ such that $\mathbb{M}_{\mathcal{J}^*}$ which doesn't add any dominating real.*
2. *$\mathcal{I}$ can be extended to a $\mathbf{P}^+$-ideal.*
3. *$\mathcal{I}$ can be extended to an $\mathbf{F}_\sigma$-ideal.*

Let $\mathcal{Z} = \{A \subset \omega : \lim_{n\to\omega} \frac{|A\cap n|}{n} = 0\}$.

Corollary

$\mathbb{M}_{\mathcal{Z}}$ adds a dominating real.

Question

Are there forcing notion which destroys $\mathcal{Z}$ and doesn't add dominating real?

Ultrafilter case

We say an ultrafilter $\mathcal{U}$ is **Canjar** if $\mathcal{U}^{<\omega}$ is P^+ ideal.

Question

When $\mathcal{U}$ is Canjar?

Thank you!

Reference

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Appendix: Countinuous reading of name

Definition

Let I be a σ-ideal on a Polish space such that the forcing $\mathbb{P}_I$ is proper. The forcing $\mathbb{P}_I$ has the **continuous reading of names** if for every I-positive Borel set B and a Borel function $f : B \to 2^\omega$ there is an I-positive Borel set $C \subset B$ such that $f \restriction C$ is continuous.

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Appendix: Ultrafilter

Definition

Let $\mathcal{U}$ be a filter on ω.

1. $\mathcal{U}$ is selective ultrafilter if $\forall f \in \omega^\omega \exists U \in \mathcal{U}(f \restriction U$ is one-to-one or constant$)$.
2. $\mathcal{U}$ is nowhere dense ultrafilter if $\forall f : \omega \to 2^\omega \exists U \in \mathcal{U}(F[U]$ is nohere dense$)$.
3. $\mathcal{U}$ is rapid if $\forall f \in \omega^\omega \exists U \in \mathcal{U}(|U \cap f(n)| \leq n)$.
4. $\mathcal{U}$ is P-point ultrafilter if $\forall f \in \omega^\omega \exists U \in \mathcal{U}(f \restriction U$ is finite-to-one or constant$)$.

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