All automorphisms of all Calkin algebras
Part III: We actually know very little

Ilijas Farah

York University

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The story so far

Theorem (Phillips–Weaver, Farah, 2007)

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Does TA+MA imply all automorphisms of each $\mathcal{C}_\kappa$ are inner?
Does TA imply all automorphisms of $\mathcal{C}_{\aleph_1}$ are inner?

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Question

Does TA+MA imply all automorphisms of each $\mathcal{C}_\kappa$ are inner?
Does TA imply all automorphisms of $\mathcal{C}_{\aleph_1}$ are inner?
Does '$\mathcal{C}_{\aleph_0}$ has an outer automorphism' imply '$\mathcal{C}_{\aleph_1}$ has an outer automorphism?'

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TA+MA imply all automorphisms of $\mathcal{C}_{\aleph_1}$ *are inner.*
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Does TA+MA imply all automorphisms of each $\mathcal{C}_\kappa$ *are inner?*
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Does ZFC imply all automorphisms of each $\mathcal{C}_\kappa$*, for* κ *uncountable, are inner?*

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Does TA imply all automorphisms of $\mathcal{C}_{\aleph_1}$ are inner?
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Does ZFC imply all automorphisms of each $\mathcal{C}_\kappa$, for κ uncountable, are inner?

Remark

'$\mathcal{C}_{\aleph_1}$ has an outer automorphism' is a Σ^2_2 statement.

Calkin beyond separable

Lemma

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$$|\,\mathrm{Card} \cap [\aleph_0, \kappa]|.$$

Proof.

Given an infinite $\lambda \leq \kappa$ we have

$$\mathcal{K}_{\kappa,\lambda} = \overline{\{a\colon a[\ell_2(\kappa)] \text{ has density } < \lambda\}}.$$

□

A curiosity

It is an open problem whether $\mathcal{P}(\aleph_0)/\operatorname{Fin}$ and $\mathcal{P}(\aleph_1)/\operatorname{Fin}$ can be isomorphic. The 'quantized' version is not difficult.

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Lemma

$\mathcal{C}_{\aleph_0} \not\cong \mathcal{C}_{\aleph_1}$.

Proof.

$\mathcal{C}_{\aleph_0}$ is simple and $\mathcal{C}_{\aleph_1}$ isn't. □

Some outer automorphisms

For an infinite cardinal κ consider $\mathcal{D}_\kappa = \mathcal{B}_\kappa / \mathcal{K}_{\kappa,\kappa}$.

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If $2^\kappa = \kappa^+$ and κ is regular then $\mathcal{D}_\kappa$ has 2^{κ^+} automorphisms, hence an outer automorphism.

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It suffices to assume that the club filter on κ is κ^+-generated and $2^{\kappa^+} > 2^\kappa$.
(Cf. with the fact that $\mathcal{C}_{\aleph_0}$ has an outer automorphism if $\mathfrak{d} = \aleph_1$ and $2^{\aleph_1} > 2^{\aleph_0}$.)

Proof of the case $\kappa = \aleph_1$

For a club $\mathbf{C} \subseteq \aleph_1$ let

$$\mathcal{D}[\mathbf{C}] = \{a \in \mathcal{B}_{\aleph_1} : ap_\xi = p_\xi a \text{ for all } \xi \in \mathbf{C}\}.$$

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Proof.

For $a \in \mathcal{B}_{\aleph_1}$ fix a continuous ϵ-chain M_ξ, $\xi < \omega_1$, of elementary submodels of $H_{\aleph_2}$ containing a. Their intersections with $\aleph_1$ define $\mathbf{C}$ such that $a \in \mathcal{D}[\mathbf{C}]$. □

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Let $\Phi = \operatorname{Ad} u$.

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Let $\Phi = \mathrm{Ad}\, u$.

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For $a \in \mathcal{D}[\mathbf{C}]$ we have $(I - 2p)a(I - 2p) = a$, hence Ψ_1 and Ψ_2 agree on $\mathcal{D}[\mathbf{C}]$.

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For $a \in \mathcal{D}[\mathbf{C}]$ we have $(I - 2p)a(I - 2p) = a$, hence Ψ_1 and Ψ_2 agree on $\mathcal{D}[\mathbf{C}]$.
If $\pi(p) \notin Z(\mathcal{D}[\mathbf{C}])/ \pmod{\mathcal{K}_{\aleph_1,\aleph_1}}$ then Ψ_1 and Ψ_2 are as required. □

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$\dots$	X	X	X	?

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$\lambda \setminus \kappa$	$\aleph_0$	$\aleph_1$	$\aleph_2$	$\dots$
$\aleph_0$	TA	TA+MA	PFA	PFA
$\aleph_1$	X	?	?	?
$\aleph_2$	X	X	?	?
$\dots$	X	X	X	?

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$\dots$	X	X	X	GCH

Question

What is the 'right' statement of the Rigidity Conjecture?